Uncertainty of Inter-/Extrapolation and Averaging

Introduction

In order to evaluate the uncertainty of the interpolation, extrapolation and averaged SAR calculation algorithms of the Postprocessor, DASY5 allows the generation of measurement grids which are artificially predefined by analytically based test functions. Therefore, the grids of area scans and zoom scans can be filled with uncertainty test data, according to the SAR benchmark functions of [1].The three analytical functions shown in equation (19.6) are used to describe the possible range of the expected SAR distributions for the tested handsets. The field gradients are covered by the spatially flat distribution f_1, the spatially steep distribution f_3 and f_2 accounts for H-field cancellation on the phantom/tissue surface.

$$
\begin{aligned}
f_1(x,y,z) &= Ae^{-\frac{z}{2a}}\cos^2\left(\frac{\pi}{2}\frac{\sqrt{x'^2+y'^2}}{5a}\right)\\
f_2(x,y,z) &= Ae^{-\frac{z}{a}}\frac{a^2}{a^2+x'^2}\left(3-e^{-\frac{2z}{a}}\right)\cos^2\left(\frac{\pi}{2}\frac{y'}{3a}\right)\\
f_3(x,y,z) &= A\frac{a^2}{\frac{a^2}{4}+x'^2+y'^2}\left(e^{-\frac{2z}{a}}+\frac{a^2}{2(a+2z)^2}\right)
\end{aligned}
\tag{19.6}
$$

For the 1g and 10g reference SAR values given by the distribution functions shown in equation (19.7), the value of A was arbitrarily chosen as $1.0\,W/kg$ and the delay factor of a was arbitrarily set as $20\,mm$. These SAR values are calculated to a precision of $0.01\,\%$.

$$
\begin{aligned}
SAR_{ref} &= (f_1)_{1g} = 0.881\ W/kg, \quad SAR_{ref} = (f_1)_{10g} = 0.758\ W/kg\\
SAR_{ref} &= (f_2)_{1g} = 1.796\ W/kg, \quad SAR_{ref} = (f_2)_{10g} = 1.375\ W/kg\\
SAR_{ref} &= (f_3)_{1g} = 3.048\ W/kg, \quad SAR_{ref} = (f_3)_{10g} = 1.385\ W/kg
\end{aligned}
\tag{19.7}
$$

Using the same values for A and a as described above, the corresponding reference peak SAR values at the location $x = y = z = 0.0$ for the three functions are as indicated in equation (19.8).

$$
\begin{aligned}
SAR_{ref} &= (f_1)_{peak} = 1\ W/kg\\
SAR_{ref} &= (f_2)_{peak} = 2\ W/kg\\
SAR_{ref} &= (f_3)_{peak} = 6\ W/kg
\end{aligned}
\tag{19.8}
$$

Evaluation of Maximum Search

The area scan evaluation must result in a determination of the local maximum with a precision of better than half of the side length of the zoom scan, i.e., better than 15 mm assuming the zoom scan has a side length of larger than 30 mm. Table 19.2 indicates the uncertainty of the Area Scan interpolation, when shifting the measurement grid of the Area Scan around the maximum location of the analytical functions f_1, f_2, and f_3. By application of a measurement grid resolution of 20 mm and a resampling of the interpolation by 2 mm, the calculation of the maximum location by deviations ≤ 2 interpolation grid steps is demonstrated, i.e., less than 4 mm displacement. For 30 mm, the interpolation tolerance is still within the requirements. In conclusion, the interpolation scheme implemented in DASY5 fulfills the requirements for grid steps of < 20 mm for the area scan.

Displacement $(\Delta x, \Delta y)$ in mm	Function f_1 $(\Delta x, \Delta y)$ in mm	Function f_2 $(\Delta x, \Delta y)$ in mm	Function f_3 $(\Delta x, \Delta y)$ in mm
$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$
$(2,0)$	$(0,0)$	$(0,0)$	$(2,0)$
$(4,0)$	$(0,0)$	$(2,0)$	$(2,0)$
$(6,0)$	$(0,0)$	$(2,0)$	$(4,0)$
$(8,0)$	$(0,0)$	$(2,0)$	$(4,0)$
$(10,0)$	$(0,0)$	$(0,0)$	$(0,0)$
$(2,2)$	$(0,0)$	$(0,0)$	$(2,-2)$
$(4,4)$	$(0,0)$	$(2,0)$	$(2,-2)$
$(6,6)$	$(0,0)$	$(2,0)$	$(4,-4)$
$(8,8)$	$(0,0)$	$(0,0)$	$(2,-2)$
$(10,10)$	$(0,0)$	$(0,-2)$	$(0,0)$

Table 19.2: Area scan shifts: the peak value position of the test functions f_1, f_2, and f_3 are shifted by the displacement vector. The resolution of the grid of the area scan is 20mm, whereby an interpolation resolution of 2mm is applied. The worst-case situation occurs for a shift of a quarter of an area scan grid step. The Area Scan interpolation scheme determines correct positions for every shift distance with respect to function f_1. On the basis of functions f_2 and f_3, a good calculation of the maximum location by deviations ≤ 2 interpolation grid steps is still demonstrated.

Evaluation of Tolerance

Tables 19.3 and 19.2 summarize the evaluated uncertainty of the Zoom Scan interpolation, extrapolation as well as Av. SAR algorithm and the determination of the maxima locations in the Area Scan, respectively.The 1 g, 10 g averaged SAR and peak SAR uncertainties indicated in Table 19.3 are expressed according to the relationship in equation (19.9).

$$SAR_{tolerance}[\%] = 100 \times \left| \frac{SAR_{evaluated} - SAR_{ref}}{SAR_{ref}} \right| \tag{19.9}$$

In Table 19.3, the deviations for the spatial peak SAR averaged over 1 g and 10 g are summarized for different lateral shifts of the zoom scans (7(Δ=5 mm) x 7 (Δ=5 mm) x 7 (Δ=5 mm) assuming the sensor center is at 4 mm distance from the surface. The maximum tolerance is less than 0.7%. For a 5 mm sensor distance and an enlarged grid step (5(Δ=9 mm) x 5 (Δ=9 mm) x 7 (Δ=5 mm), the tolerance is still better than 4.5% with greatly improved evaluation speed. In conclusion, the extrapolation, interpolation and averaging uncertainty is less than 1.0% (rectangular distribution) provided the sensor distance is between 4 and 5 mm.

displ. $(\Delta x, \Delta y)$ in mm	Function f_1 SAR(1g) in %	SAR(10g) in %	Peak SAR in %	Function f_2 SAR(1g) in %	SAR(10g) in %	Peak SAR in %	Function f_3 SAR(1g) in %	SAR(10g) in %	Peak SAR in %
$(0,0)$	0.114	-0.785	-0.700	0.390	-0.434	1.900	0.656	-0.503	0.467
$(1,0)$	0.114	-0.785	-0.700	0.390	-0.362	1.950	0.689	-0.503	0.250
$(2,0)$	0.114	-0.785	-0.700	0.390	-0.362	1.950	0.755	-0.503	-0.100
$(3,0)$	0.114	-0.785	-0.700	0.390	-0.362	1.950	0.722	-0.503	-0.083
$(4,0)$	0.114		-0.700	0.445		2.050	0.623		0.283
$(5,0)$	0.114		-0.600	0.445		2.100	0.558		0.417
$(6,0)$	0.227		-0.600	0.445		2.100	0.623		0.100
$(7,0)$	0.227		-0.600	0.445		2.100	0.722		-0.333
$(8,0)$	0.227		-0.600	0.501		2.150	0.787		-0.317
$(0,1)$	0.114	-0.785	-0.700	0.390	-0.434	1.900	0.689	-0.503	0.250
$(0,2)$	0.114	-0.785	-0.700	0.390	-0.434	1.900	0.755	-0.503	-0.100
$(0,3)$	0.114	-0.785	-0.700	0.390	-0.362	1.950	0.722	-0.503	-0.083
$(0,4)$	0.114		-0.700	0.390		1.950	0.623		0.283
$(0,5)$	0.114		-0.600	0.390		2.000	0.558		0.417
$(0,6)$	0.227		-0.600	0.390		2.000	0.623		0.083
$(0,7)$	0.227		-0.600	0.390		2.000	0.722		-0.333
$(0,8)$	0.227		-0.600	0.445		2.050	0.787		-0.317
$(1,1)$	0.114	-0.785	-0.700	0.390	-0.362	1.950	0.722	-0.503	0.050
$(2,2)$	0.114	-0.785	-0.700	0.390	-0.362	1.950	0.853	-0.431	-0.567
$(3,3)$	0.227	-0.785	-0.700	0.445	-0.362	2.000	0.820	-0.431	-0.583
$(4,4)$	0.227		-0.600	0.445		2.100	0.591		-0.117
$(5,5)$	0.227		-0.600	0.445		2.150	0.459		-0.017
$(6,6)$	0.227		-0.600	0.445		2.200	0.558		-0.733
$(7,7)$	0.227		-0.500	0.501		2.250	0.820		-1.417
$(8,8)$	0.227		-0.500	0.501		2.300	1.017		-1.233

Table 19.3: Error of analytical solution in percent. If $\Delta x > 8$ and/or $\Delta y > 8$, the measured cube is toosmall for the SAR(1g). If $\Delta x > 3$ and/or $\Delta y > 3$, the measured cube is toosmall for the SAR(10g). Thus, in the table there is an empty element.