Microwave RF power density measurement
for transmitters with high-gain antennas

Background

Regulatory limits for RF power density, in terms of microwatts/m^2 and similar units, are established under subparts 15.253 and 15.255 of FCC Part 15 Rules. According to Sanders, NTIA equation C.7b, far-field power density can be predicted from the field strength E (V/m) and the far-field impedance of free space 120π or 377Ω, from the basic equation $P = V^2/R$ as

(1) $$S(W/m^2) = E(V/m)^2/377$$

or, when the transmitter power in ERP is known, from the equation given in OET 65 page 21,

(2) $$S(\mu W/cm^2) = 33.4\ ERP/R^2$$

where R = the distance in meters from the transmitter antenna to the measuring antenna.

Subparts 15.253 and 15.255 set the power density limits at a measurement distance of 3m. For high-gain antennas used at microwave frequencies, the 3m distance is likely to be within the antenna's near field or Fresnel region. This can be confirmed from the equation for near-field boundary given in OET 65 (p. 27):

(3) $$R_{nf} = D^2/4\lambda$$

For a parabolic reflector (Cassegrain) antenna with D = 30cm and λ = 0.5 cm (about 60 GHz), equation (3) predicts a near-field boundary of R_{nf} = 4.5m.

The corresponding far-field boundary is given in OET 65 (page 29) as:

(4) $$R_{ff} = 0.6\ D^2/\lambda$$

For the same parameters as above, equation (4) predicts a far-field or plane-wave boundary of R_{ff} = 10.8m.

It is important to know where the near-field and far-field boundaries exist with respect to the 3m measuring distance, because the validity of equations (1) and (2) break down in the near field. OET 65, page 46 notes:

> "…a readout device calibrated in units of power density does not read true power density if measurements are made in the near-field. This is because under plane-wave conditions, in which E, H, and power density are related by a constant quantity (the wave impedance which, for free space, is equal to 377 ohms), do not exist in the near-field where the wave impedance is complex and generally not known."

Fortunately, it is possible to predict power densities S in the near and far fields, as well as in the "transition region" in between, as a function of the transmitter output power P and distance from the transmitting antenna R. The equations are given in OET 65 pages 27 – 29 as follows:

(5) at the surface of the antenna with aperture area A: $S_{surface} = 4P/A$

(6) in the near field, with antenna maximum dimension D: $S_{nf} = 16\eta P/\pi D^2$, η = aperture efficiency

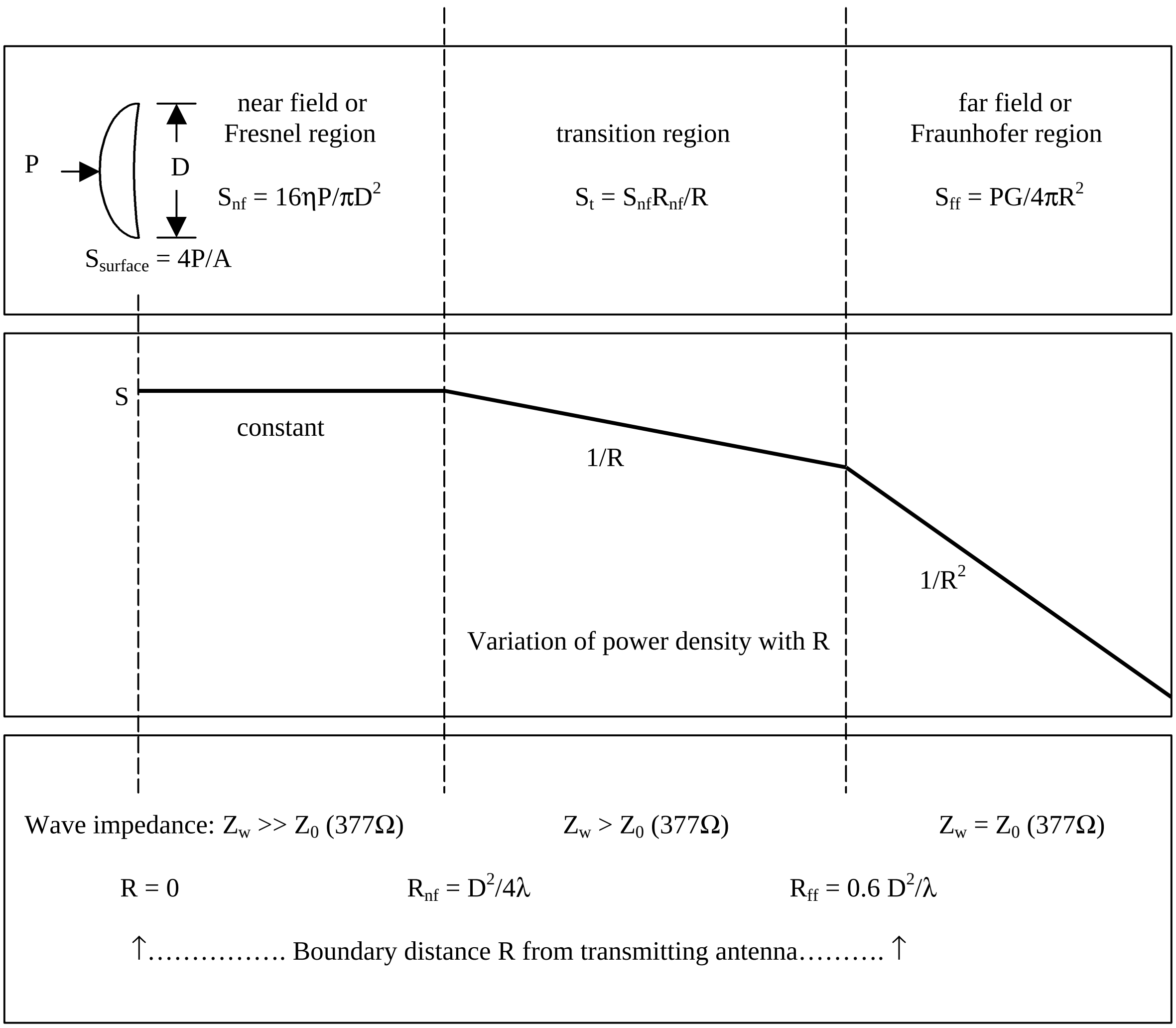
(7) in the transition region $R_{nf} < R < R_{ff}$ at distance R: $S_t = S_{nf}R_{nf}/R$

(8) in the far field or Fraunhofer region: $S_{ff} = PG/4\pi R^2$, G = transmitter antenna gain

The equations (5) – (8) indicate that the variation of power density with distance R from the transmitter antenna is:

a) constant from the antenna surface to the near-field boundary R_{nf};
b) decreases as 1/R in the transition region $R_{nf} < R < R_{ff}$;
c) decreases as $1/R^2$ in the far field, for $R > R_{ff}$.

The equations and power density relationships are summarized and illustrated graphically in the diagram below.



In cases where the measurement distance of 3m is closer to the transmitting antenna than the Fraunhofer region or the far field, the assumption a) that wave impedance is 377Ω, and b) the equation relating field strength and power density (1) are not valid.

The following analysis presents a method for estimating the power density at 3m when the far-field boundary may be > 3m. It should be supported by measurements of field strength at points intermediate between 3m and the far-field boundary.

Methodology

Where power density varies with distance from a transmitting antenna by an exponent n (n = 0, 1 or 2 as indicated above), the relationship between power densities at two points within the same region (near field, transition or far field) is given by the ratio:

(9) $S_1/S_2 = (R_2)^n/(R_1)^n$

(10) or $S_1 = S_2\,(R_2)^n/(R_1)^n$

(11) taking the common logarithm, $\log_{10}S_1 = \log_{10}S_2 + n \log_{10} (R_2/R_1)$

(12) as dB power is 10 log(P), we have $10 \log_{10}S_1 = 10 \log_{10}S_2 + 10\, n \log_{10} (R_2/R_1)$

(13) or $\text{dB } S_1 = \text{dB } S_2 + 10\, n \log_{10} (R_2/R_1)$

(14) so that for n = 1 the equation becomes $\text{dB } S_1 = \text{dB } S_2 + 10 \log_{10} (R_2/R_1)$

(15) and for n = 2 the equation is $\text{dB } S_1 = \text{dB } S_2 + 20 \log_{10} (R_2/R_1)$

Equations for power density

Case 1: R_{ff} < 3m The far field boundary is closer than 3m to the transmitting antenna. The measurement of field strength E can be made at 3m, and the assumption of wave impedance = 377Ω is valid.

$\text{dB } S_3 = 10 _{\log10} [E^2/377]$ or

(16) $\text{dB } S_3\,(\mu\text{W/cm}^2) = E_3\ (\text{dB}\mu\text{V/m}) – 125.76$

Case 2: R_{ff} > 3m > R_{nf} The specified measurement distance of 3m lies inside of the transition region. The assumption of wave impedance = 377Ω is not valid. The field strength E will be measured in the far field at distance R and calculated for the 3m distance.

(17) $\text{dB } S_3\,(\mu\text{W/cm}^2) = \text{dB } S_R\,(\mu\text{W/cm}^2) + 20 \log_{10} (R/R_{ff}) + 10 \log_{10} (R_{ff}/3\text{m})$

Case 3: R_{nf} > 3m The specified measurement distance of 3m lies inside of the near field. The assumption of wave impedance = 377Ω is not valid. The field strength E will be measured in the far field at distance R and calculated for the 3m distance.

(18) $\text{dB } S_3\,(\mu\text{W/cm}^2) = \text{dB } S_R\,(\mu\text{W/cm}^2) + 20 \log_{10} (R/R_{ff}) + 10 \log_{10} (R_{ff}/R_{nf})$

In the equation above, the term (R_{ff}/R_{nf}) can be further simplified because, from equations (3) and (4), the ratio (R_{ff}/R_{nf}) has a constant value of 2.4 for the same values of antenna size and wavelength. Thus equation (18) becomes:

(19) $\text{dB } S_3\,(\mu\text{W/cm}^2) = \text{dB } S_R\,(\mu\text{W/cm}^2) + 20 \log_{10} (R/R_{ff}) + 3.8$